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Model Reference Control of Distributed Parameter Systems: Application to the SCOLE Problem

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MODEL REFERENCE CONTROL OF DISTRIBUTED PARAMETER SYSTEMS WITH APPLICATION TO THE SCOLE PROBLEM

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OUTLINE

- INTRODUCTION
- MODEL REFERENCE CONTROL OF LUMPED LINEAR SYSTEMS

  THEORY

  SCOLE APPLICATION
- MODEL REFERENCE CONTROL OF DPS

  THEORY

  SCOLE APPLICATION
- CONCLUSIONS AND RECOMMENDATIONS

INTRODUCTION

SCOLE MODELS

- LUMPED: 16th ORDER WITH 5 FLEXIBLE AND 3 RIGID BODY MODES
- DISTRIBUTED: 3 PARTIAL DIFFERENTIAL EQUATIONS FOR ROLL, PITCH, YAW BEAM BENDING

LUMPED MODEL

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$x^T = (\underline{u}_1^T, \ldots, \underline{\dot{u}}_8^T, \underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \psi_{RB}^T)$$

$$\underline{y}_F^T = (\underline{\phi}_s^T, \underline{\theta}_s^T, \underline{\psi}_s^T, \underline{\phi}_r^T, \underline{\theta}_r^T, \underline{\psi}_r^T, \zeta_x, \zeta_y)$$

$$y^T = y_F^T + (\underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \underline{\psi}_{RB}^T, \underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \underline{\psi}_{RB}^T\ 0, 0)$$

$$\underline{u}^T = (\underline{T}_s, \underline{f}_r, \underline{T}_r)$$

OBJECTIVE: IF $\phi_{RB}(0) = 20°$

$\phi_{RB} \to 0$ IN ABOUT 10 SEC.

$|T| \leq 10{,}000$

$|f| \leq 800$

DISTRIBUTED MODEL

ROLL BEAM BENDING:

$$PA \frac{\partial^2 u_\phi}{\partial t^2} + 2\zeta_\phi \sqrt{PA\ EI_\phi} \frac{\partial^3 u_\phi}{\partial s^2 \partial t} + EI_\phi \frac{\partial^4 u_\phi}{\partial s^4} = \sum_{n=1}^{4} \left[f_{\phi,n} \delta(s-s_n) + g_{\phi,n} \frac{\partial \delta}{\partial s}(s-s_n) \right]$$

PITCH BEAM BENDING:

$$PA \frac{\partial^2 u_\Theta}{\partial t^2} + 2\zeta_\Theta \sqrt{PA\ EI_\Theta} \frac{\partial^3 u_\Theta}{\partial s^2 \partial t} + EI_\Theta \frac{\partial^4 u_\Theta}{\partial s^4} = \sum_{n=1}^{4} \left[f_{\Theta,n} \delta(s-s_n) + g_{\Theta,n} \frac{\partial \delta}{\partial s}(s-s_n) \right]$$

YAW BEAM TORSION:

$$PI_\psi \frac{\partial^2 u_\psi}{\partial t^2} + 2\zeta_\psi I_\psi \sqrt{GP} \frac{\partial^3 u_\psi}{\partial s^2 \partial t} + GI_\psi \frac{\partial^2 u_\psi}{\partial s^2} = \sum_{n=1}^{4} g_{\psi,n} \frac{\partial \delta}{\partial s}(s - s_n)$$

MODEL REFERENCE CONTROL OF LUMPED LINEAR SYSTEMS

THEORY

$$\left.\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y_p &= C_p x_p \end{aligned}\right\} \text{PROCESS}$$

$$\left.\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ y_m &= C_m x_m \end{aligned}\right\} \text{REFERENCE MODEL}$$

DESIRE

$$y_p \rightarrow y_m$$

DEFINE IDEAL STATE AND CONTROL

$$\dot{x}_p^* = A_p x_p^* + B_p u_p^*$$

$$y_p^* = C_p x_p^*$$

WHERE $y_p^* = C_m x_m = y_m$

WILL FORCE $x_p \to x_p^*$

$$\Longrightarrow y_p \to y_p^* = y_m$$

ASSUME

$$x_P^* = S_{11}\, x_M + S_{12}\, u_M$$

$$u_P^* = S_{21}\, x_M + S_{22}\, u_M$$

THEN

$$S_{11}\, A_M - A_P\, S_{11} = B_P\, S_{21}$$

$$S_{11}\, B_M - A_P\, S_{12} = B_P\, S_{22}$$

$$C_P\, S_{11} = C_M$$

$$C_P\, S_{12} = 0$$

APPLY

$$u_p = u_p^* + K(y_m - y_p)$$

THEN

$$\dot{e} = (A_p - B_p K C_p)e$$

WHERE

$$e = x_p^* - x_p$$

$\therefore$ CHOOSE K TO STABILIZE $(A_p - B_p K C_p)$

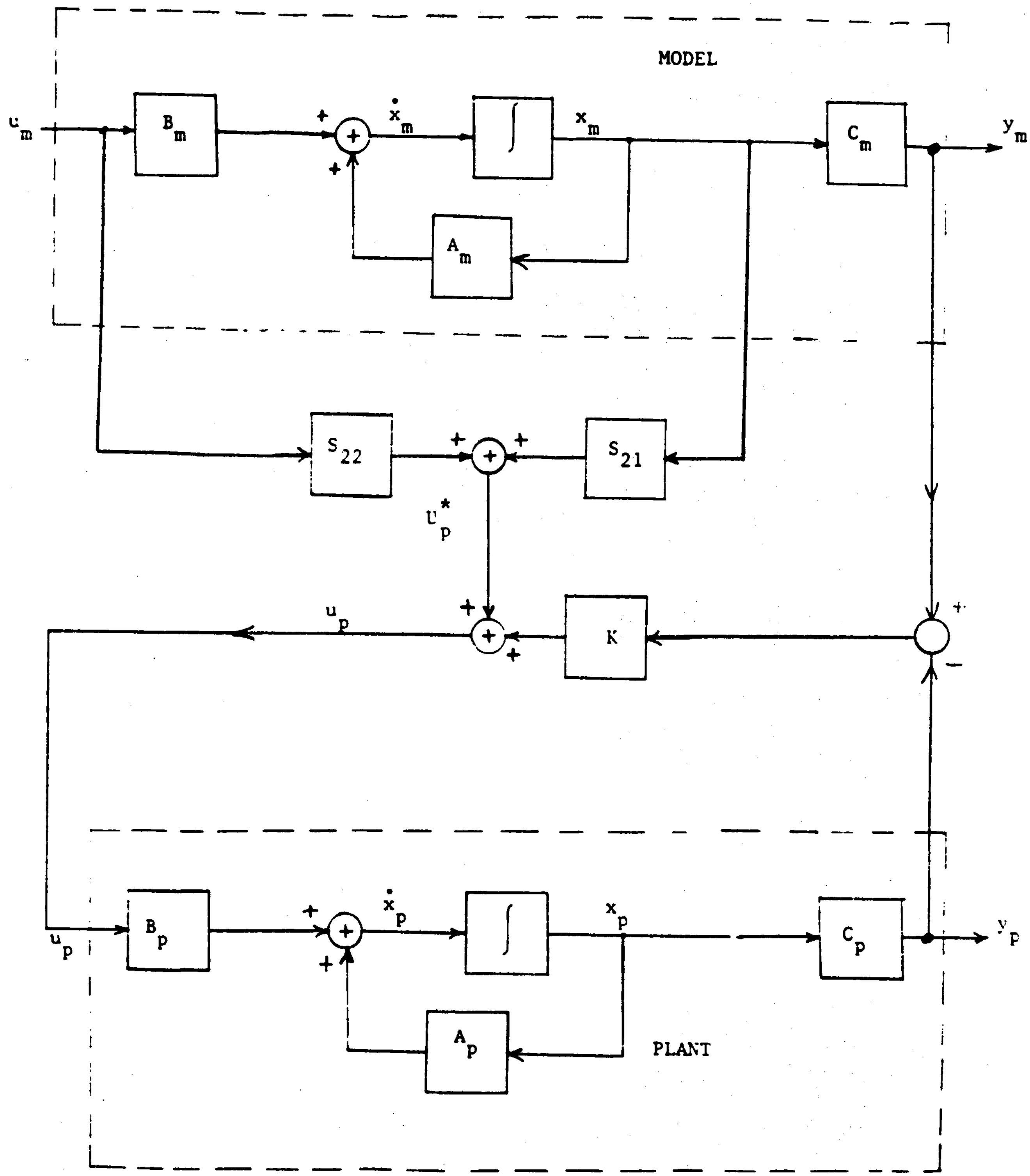


Figure 1: System block diagram.

SPECIAL CASE (PMF)

$x_p \rightarrow x_m$

OR $C_p = C_m = I$

$u_p = S_{21}\, x_m + S_{22}\, u_m + K(x_m - x_p)$

$B_p\, S_{21} = A_m - A_p$

$B_m = B_p\, S_{22}$

$(A_p - B_p\, K)$ STABLE

SCOLE APPLICATION OF LUMPED MODEL FOLLOWING

<u>OBSERVATIONS</u>

- EIGHT CONTROLS
- EIGHT OUTPUT MODES TO BE CONTROLLED

<u>PROCEDURES</u>

- PMF

  $x_p \rightarrow x_m$

- OUTPUT FOLLOWING

  $y_p \rightarrow y_m$

SPECIAL CASES: 8 outputs

CASE I: Consider only positions:

$$Y_P^T = \left[\phi_S + \phi_{RB},\ \theta_S + \theta_{RB},\ \psi_S + \psi_{RB},\ \xi_x,\ \xi_y,\ \phi_r + \phi_{RB},\ \theta_r + \theta_{RB},\ \psi_r + \psi_{RB}\right]^T$$

CASE II: Position and LOS Vectors

$$Y_P^T = \left[\phi_S + \phi_{RB},\ \theta_S + \theta_{RB},\ \psi_S + \psi_{RB},\ \xi_x,\ \xi_y,\ E_1,\ E_2,\ E_3\right]^T$$

E_1, E_2, E_3 - LOS VECTOR COMPONENTS

Note: $R_{LOS} = (R_{LOS})_{NOM} + \Delta$, where $\Delta = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$

$$E_1 = -130\ \theta_{RB} - 32.2\ \psi_{RB} + \xi_x - 32.2\ \psi_r - 260\ \theta_r$$

$$E_2 = 130\ \phi_{RB} + 18.75\ \psi_{RB} + \xi_y + 18.75\ \psi_r - 260\ \phi_r$$

$$E_3 = -18.75\ \theta_{RB} + 32.2\ \phi_{RB} - 18.75\ \theta_r - 32.2\ \phi_r$$

INITIAL CONDITIONS

Two sets of I.C.'s for each case

<u>CASE I (a)</u>:

Choose $\emptyset_{RBi}$, $\emptyset_{RBf}$, θ_{RBi} , θ_{RBf} so that e_{LOS} (o) = 20° , e_{LOS} (t_f) = 0°

$\Longrightarrow$ $\emptyset_{RBi} = 0.1063$ rad ; $\emptyset_{RBf} = -0.245$ rad

$\theta_{RBi} = -0.1115$ rad ; $\theta_{RBf} = -0.1115$

$t_f = 10$ sec.

$\Longrightarrow$ $Y_0^T = [0.0163 , -0.1115, 0 , 0 , 0 , 0.1063 , -0.1115 , 0]^T$

<u>STATE I.C.'s</u>:

$Y_p(o) = C_p X_p(o)$ ---- Solve for $X_p(o)$

<u>Model I.C.'s</u>

$Y_p \dashrightarrow Y_m$, $Y_p(o) = Y_m(o)$

$\Longrightarrow$ $X_m^T = [0.1063 , -.1115 , 0 , 0]^T$

<u>COMMAND</u>

$U_i(o) = 1.0$, $T > 0$, $i = 1, 2, \ldots, 8$

I.C.'s (cont.)

<u>CASE I (b)</u>:

$$\phi_{RB} = 0.34 \text{ rad} = 20^\circ$$

$$\psi_{RB} = \theta_{RB} = 0.0$$

$$\Longrightarrow Y_p^T(o) = [0.34\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 0.34\ ,\ 0\ ,\ 0\ ]^T$$

<u>Model</u>:

$$X_m^T = [0.34\ ,\ 0\ ,\ 0\ ,\ 0\ ]^T$$

<u>COMMAND</u>:

$$U_{m_i}(o) = 0.0\ ,\ T \geq 0,\ i = 1,\ldots,8$$

I. C."s (cont.)

<u>CASE II (a)</u>: Same objective as in CASE I (a)

$$Y_P^T(o) = [0.1063, -0.1115, 0, 0, 0, 14.49, 13.82, 5.51]^T$$

$$X_M^T(o) = [0.1063, -0.1115, 0, 0,]$$

$$\Longrightarrow Y_M(o) = Y_P(o)$$

$$U_{m_i} = 1.0, \; T \geq 0, \; i = 1, \ldots, 8$$

<u>CASE II(b)</u>: Same objective as in CASE I (b)

$$Y_P^T(o) = [0.34, 0, 0, 0, 0, 44.2, 10.948]^T$$

$$X_M^T = [0.34, 0, 0, 0,]^T$$

MODEL

In both cases: 4 states

$$\dot{X}_M = A_m X_M + B_M U_M$$

$$A_M = \mathrm{DIAG}\ [A_1\ \ A_2\ \ A_3\ \ A_4] = [-.15\ ,\ -0.10\ ,\ -0.10\ ,\ -0.10]^T$$

$$Y_M^T = [Y_{M1}\ \ Y_{M2}\ \dots\ Y_{M8}]^T$$

CASE I (a, b):

$$C_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CASE II (a, b):

$$C_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -130 & 0 & 0 \\ 130 & 0 & 0 & 0 \\ 32.2 & -18.75 & 0 & 0 \end{bmatrix}$$

MODEL (cont.)

Matrix BM:

Cases I (a), II (a):

$$B_M = \begin{bmatrix} -3.67\times10^{-2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.115\times10^{-2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Cases II (b), II (b):

$B_M = [\underline{0}]$

SYSTEM I.C.'s: RESULTS

CASE I (a):

$$Y_P^T\ (o) = Y_M^T\ (o) = [0.1063\ -0.1115,\ 0,\ 0,\ 0,\ 0.1063,\ -0.1115,\ 0]^T$$

$$X_P^T\ (o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.1063,\ 0,\ -0.1115,\ 0,\ 0,\ 0]^T$$

$$X_P^{*T}(o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.1063,\ -0.05269,\ -0.1115,\ 0,\ 0,\ 0]^T$$

CASE I (b):

$$Y_P^T\ (o) = Y_M^T\ (o) = [0.34,\ 0,\ 0,\ 0,\ 0,\ 0.34,\ 0,\ 0]^T$$

$$X_P^T\ (o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.34,\ 0,\ 0,\ 0,\ 0,\ 0]^T$$

$$X_P^{*T}(o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.34,\ -0.051,\ 0,\ 0,\ 0,\ 0,]^T$$

SYSTEM I.C.'s (cont.)

CASE II (a):

$Y^T(o)\ Y_M^T\ (o) = [0.1063,\ -0.1115,\ 0,\ 0,\ 0,\ 14.49,\ 13.82,\ 5.51]^T$

$X_P^T\ (o) = [0.118,\ 0,\ -0.0186,\ 0,\ -0.1087,\ 0,\ -0.0042,\ 0,\ -0.022,$
$0,\ 0.1063,\ 0,\ -0.1115,\ 0,\ 0,\ 0]^T$

$X_P^{*T}\ (o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.1063,\ -0.05269,\ -0.1115,\ 0,\ 0,\ 0]$

CASE II (b):

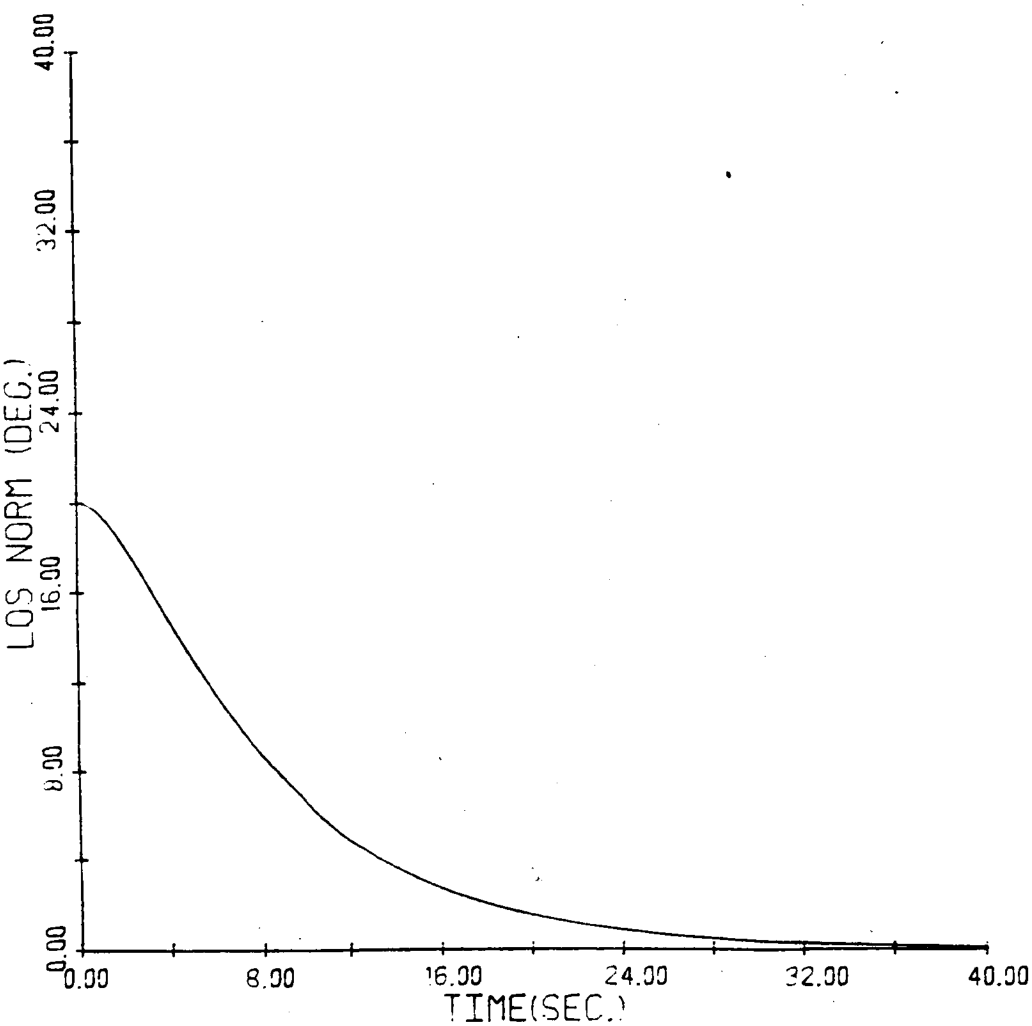
$Y_P^T\ (o) = Y_M^T\ (o) = [0.34,\ 0,\ 0,\ 0,\ 0,\ 0,\ 44.2,\ 10.948]^T$

$X_P^T\ (o)\ [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.34,\ 0,\ 0,\ 0,\ 0,]$

$X_P^*\ (o) = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.34,\ -0.051,\ 0,\ 0,\ 0,\ 0,]^T$

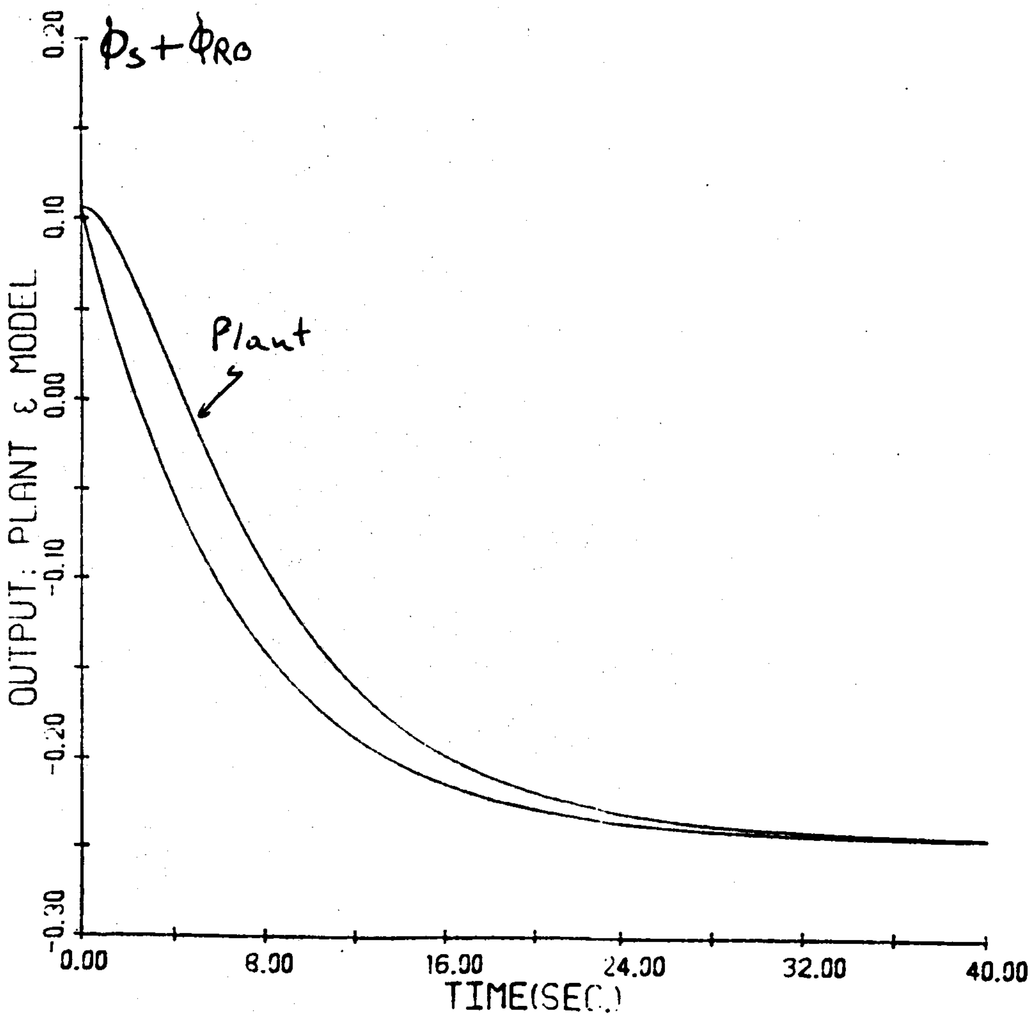
LOS ERROR : CASE II (a)

$$e_{LOS} = \sin^{-1}\left[\frac{\|\bar{\delta}_T \times T_1 \bar{R}_{LOS}\|}{\|R_{LOS}\|}\right]$$



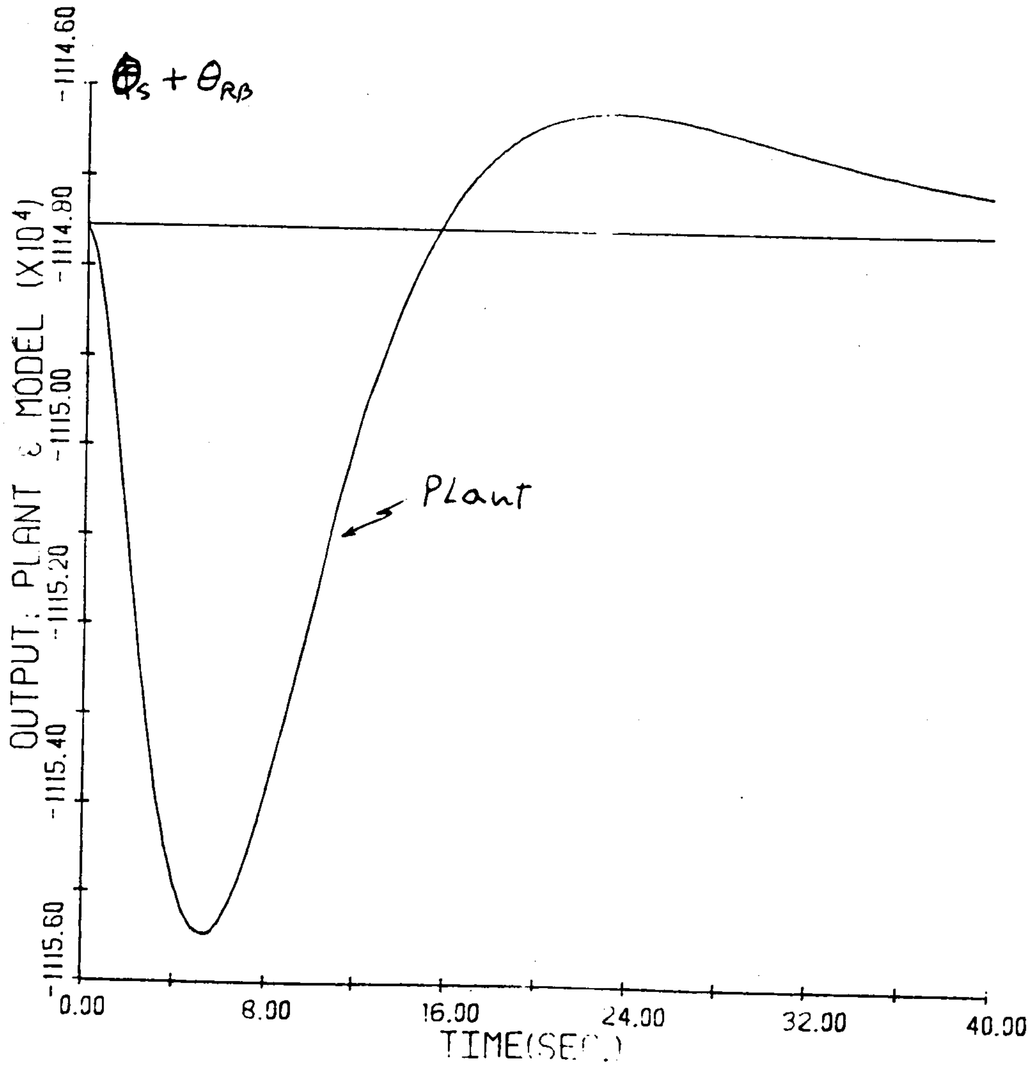
OUTPUT : PLANT and Model

Case II (a)

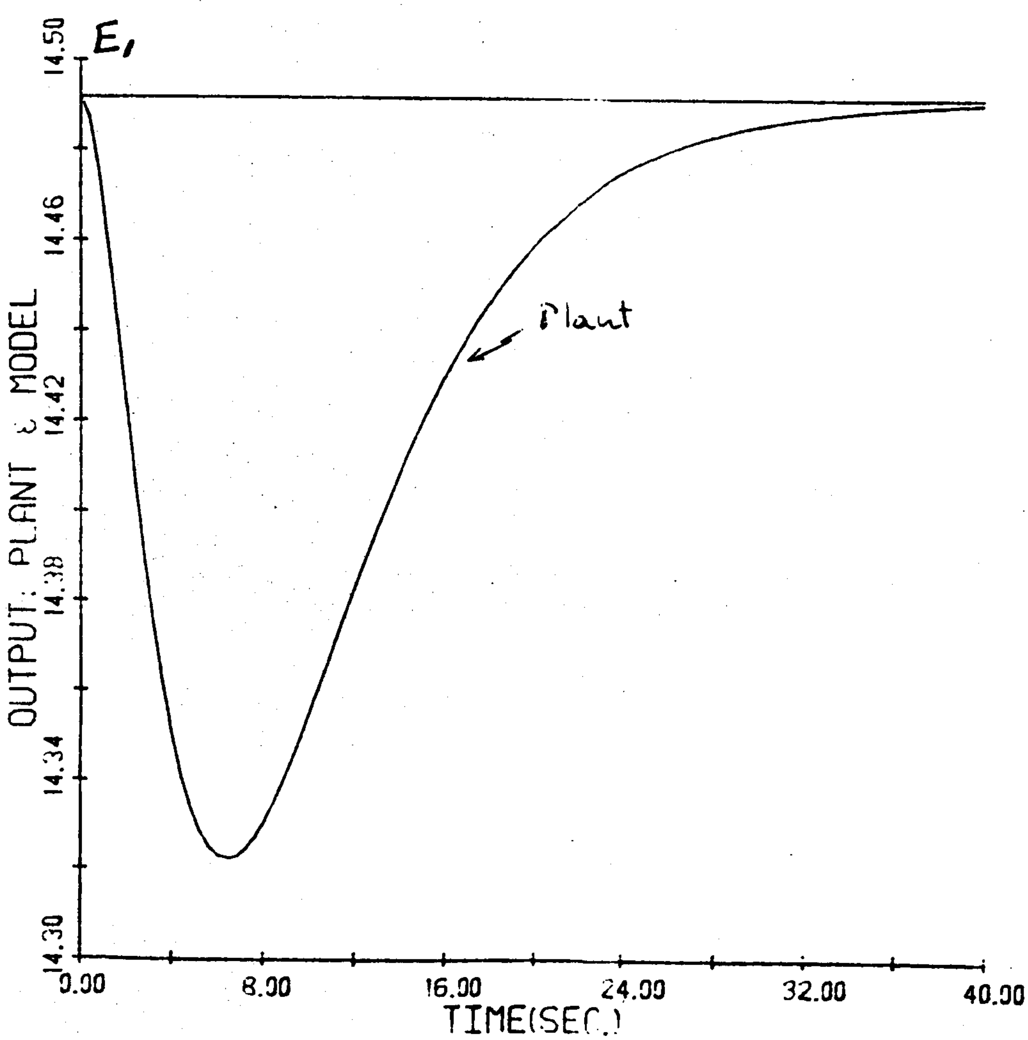


OUTPUT: PLANT and Model

Case II (a)

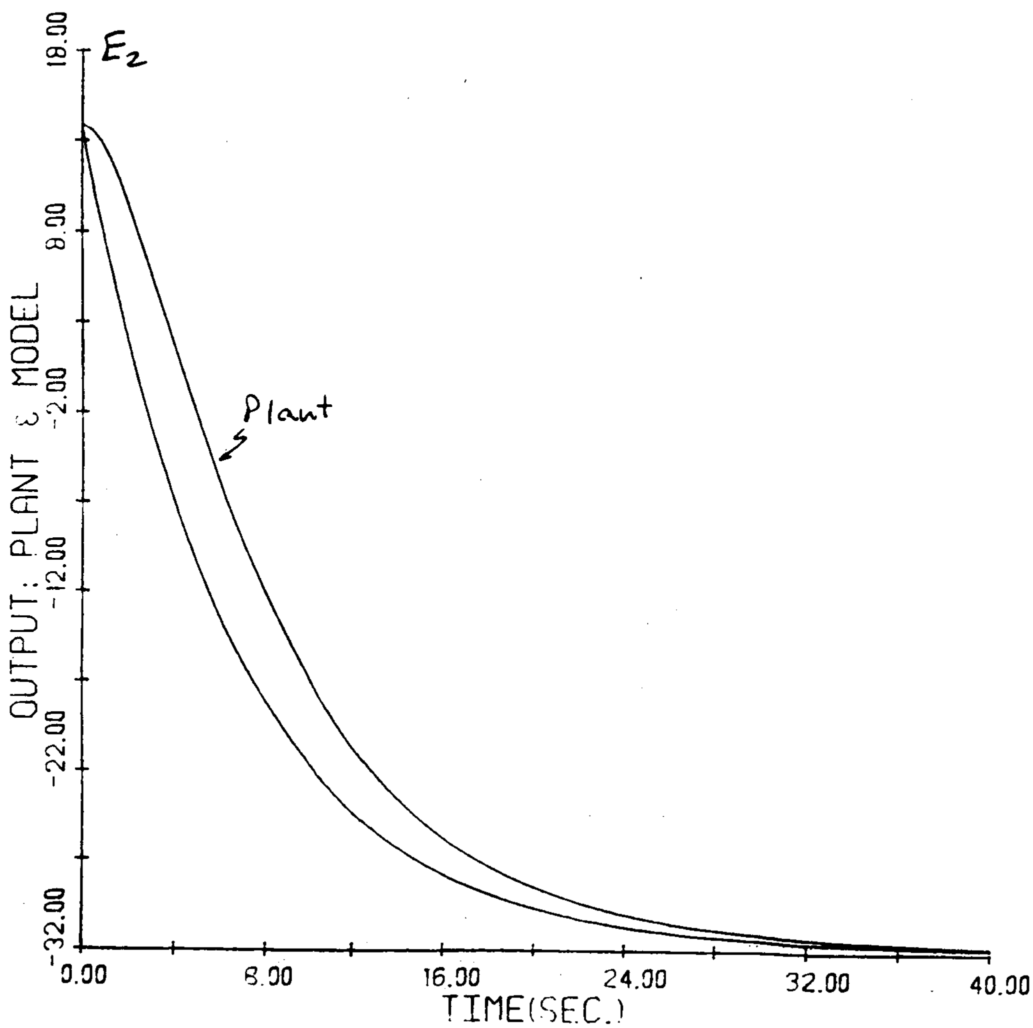


OUTPUT : PLANT and Model
CASE II (a): 1st Component of LOS



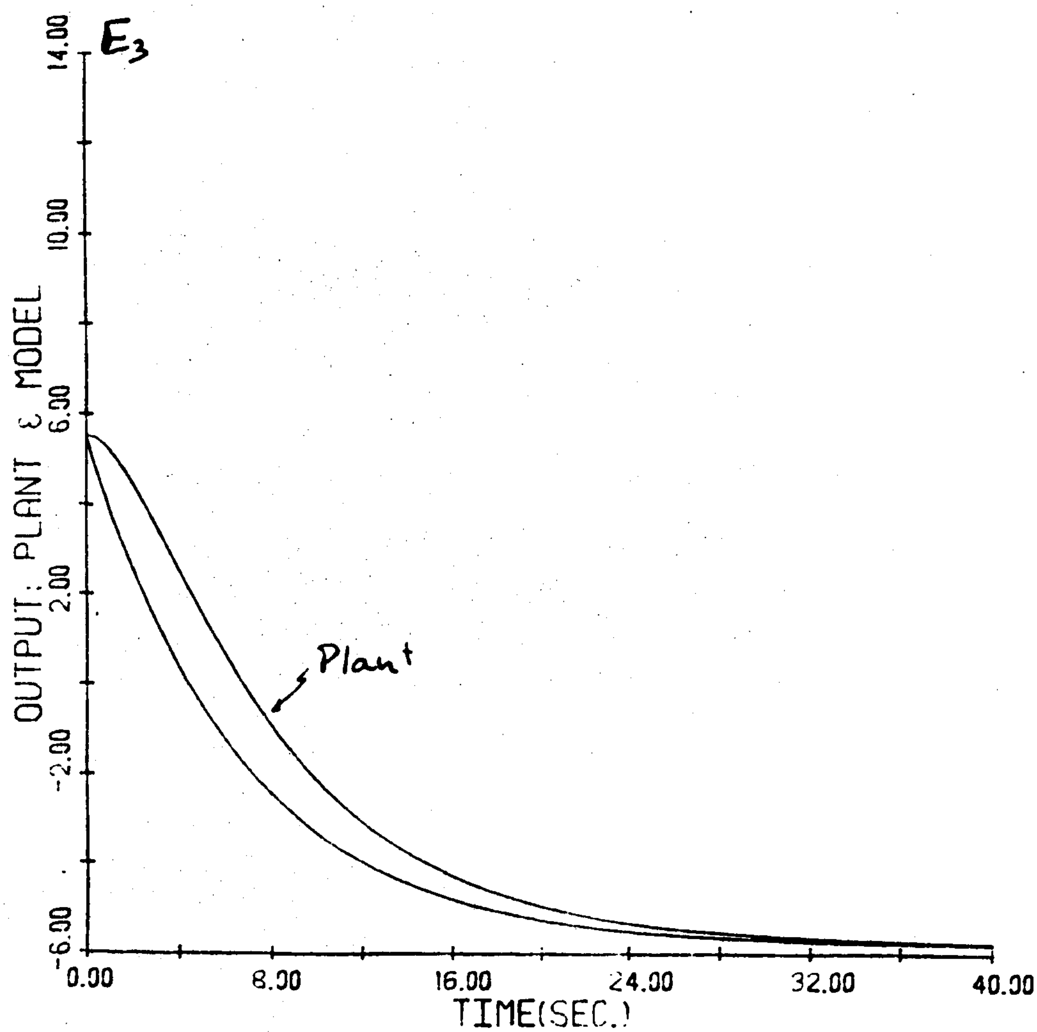
C-3

OUTPUT : PLANT and Model
CASE II (a) : 2ND Component of LOS

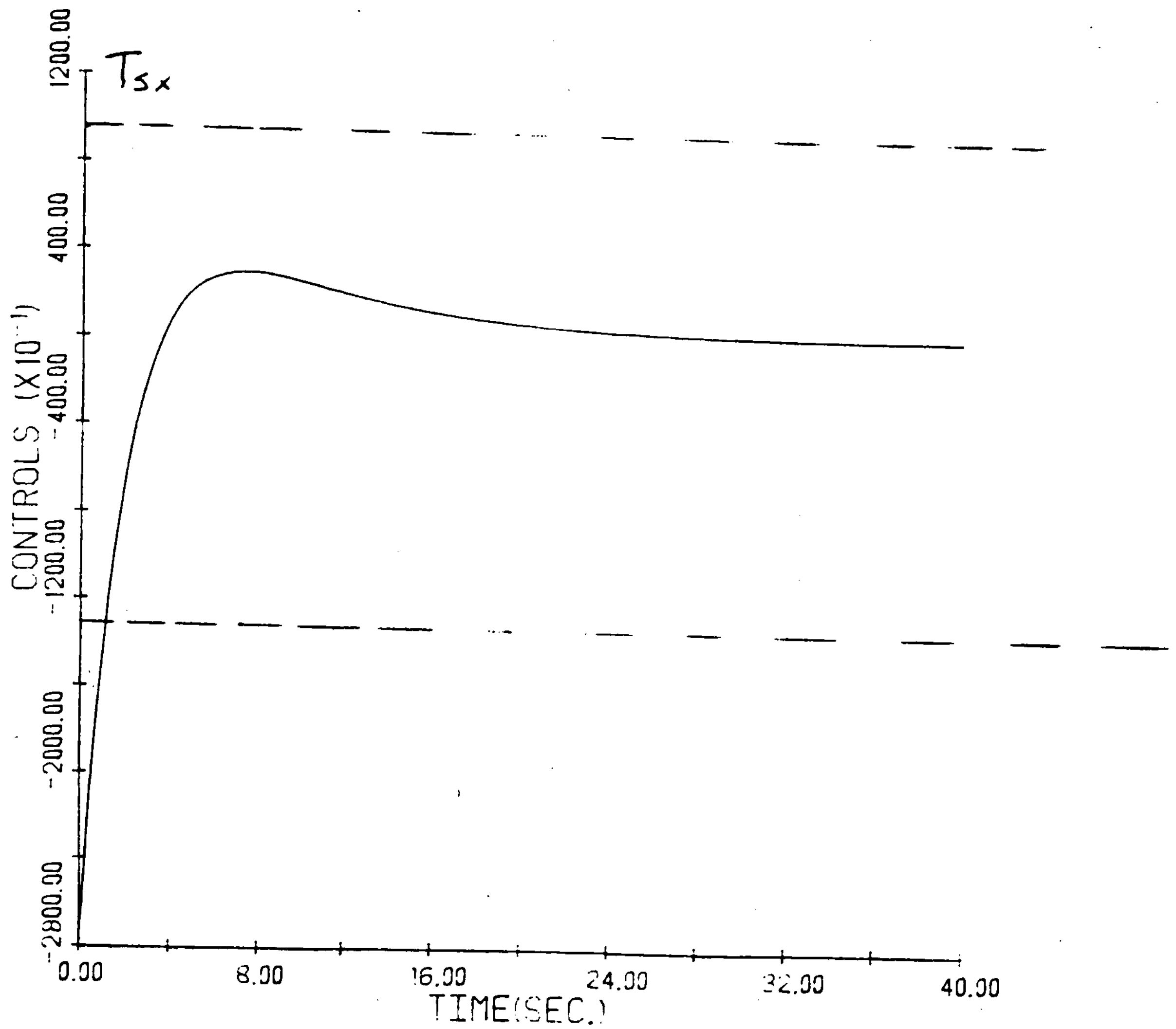


OUTPUT : PLANT and Model

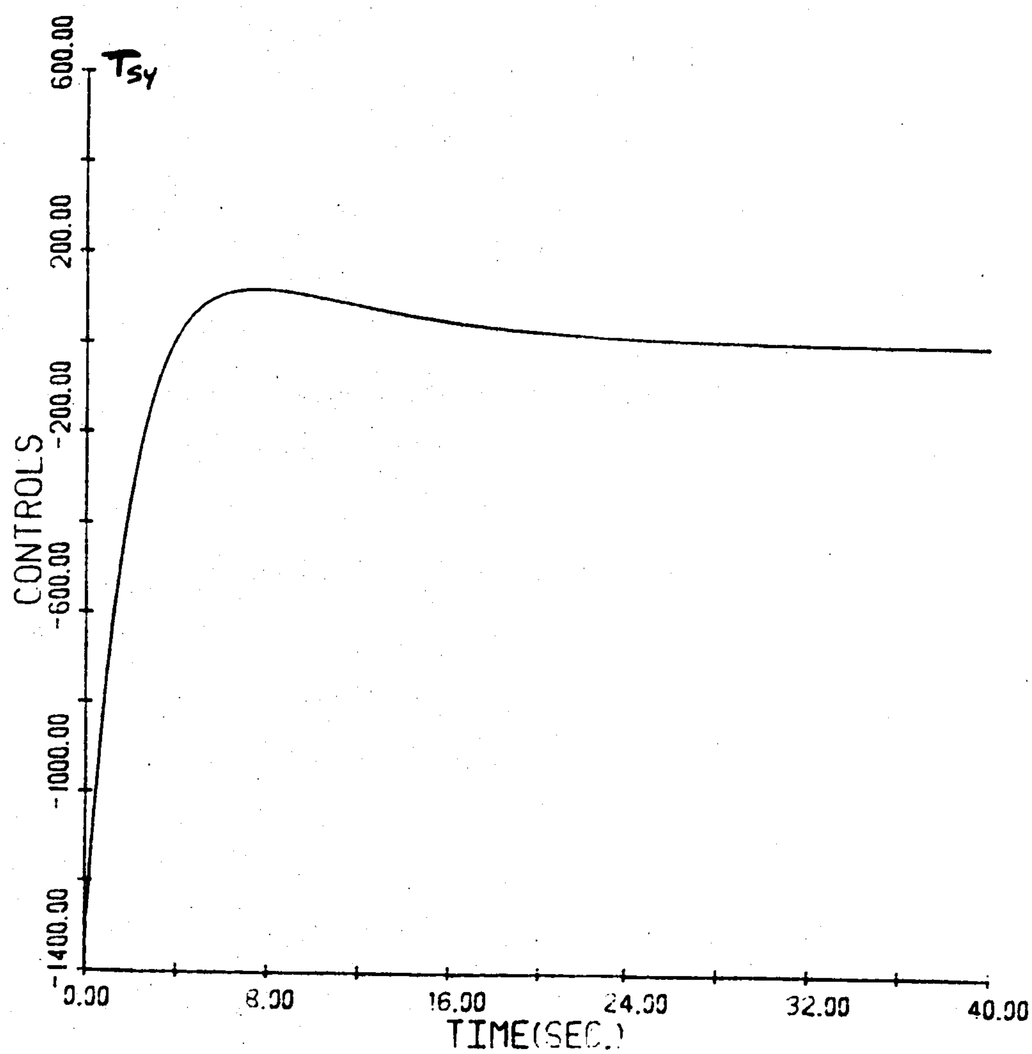
CASE II(a) : 3RD Component of LOS



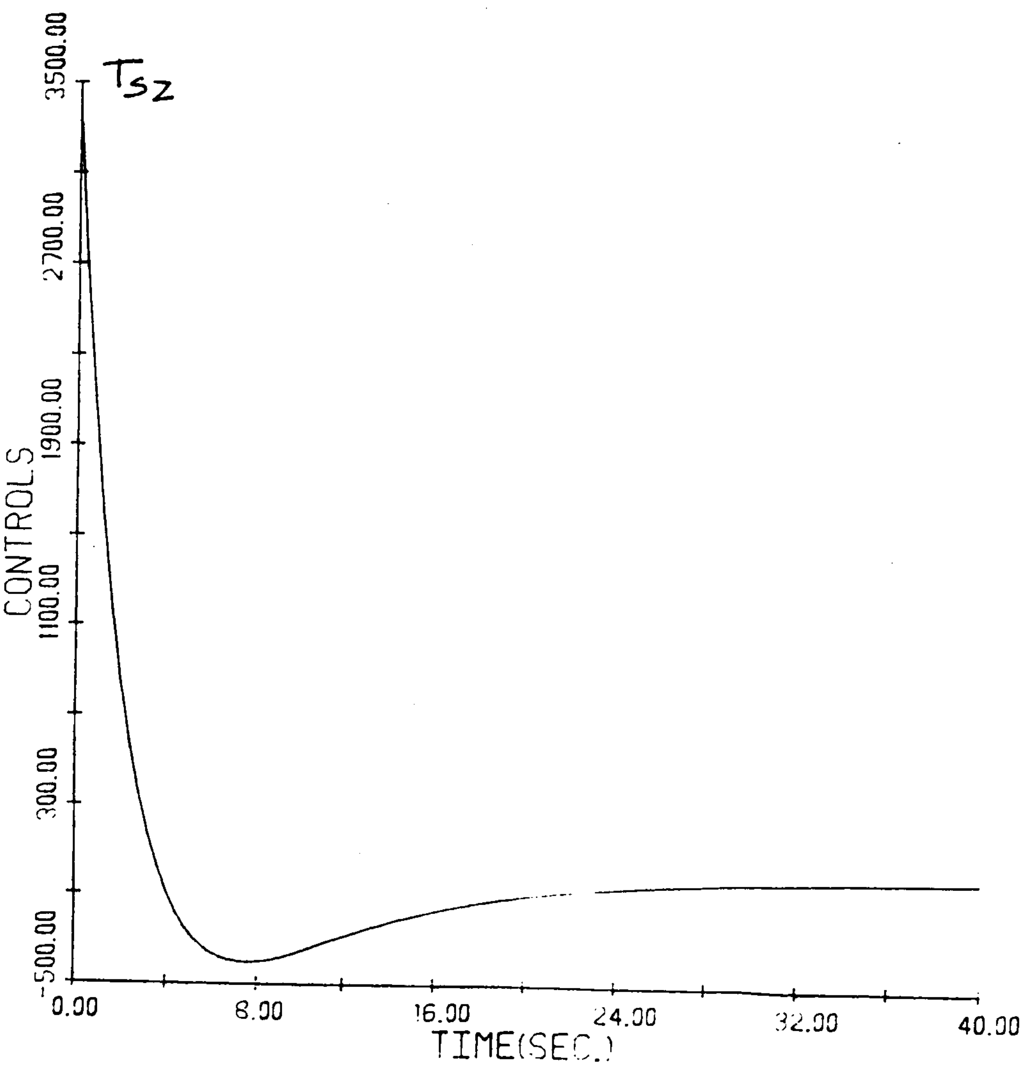
CONTROL : CASA II (a)



CONTROL : CONTROL : CASE II (a)

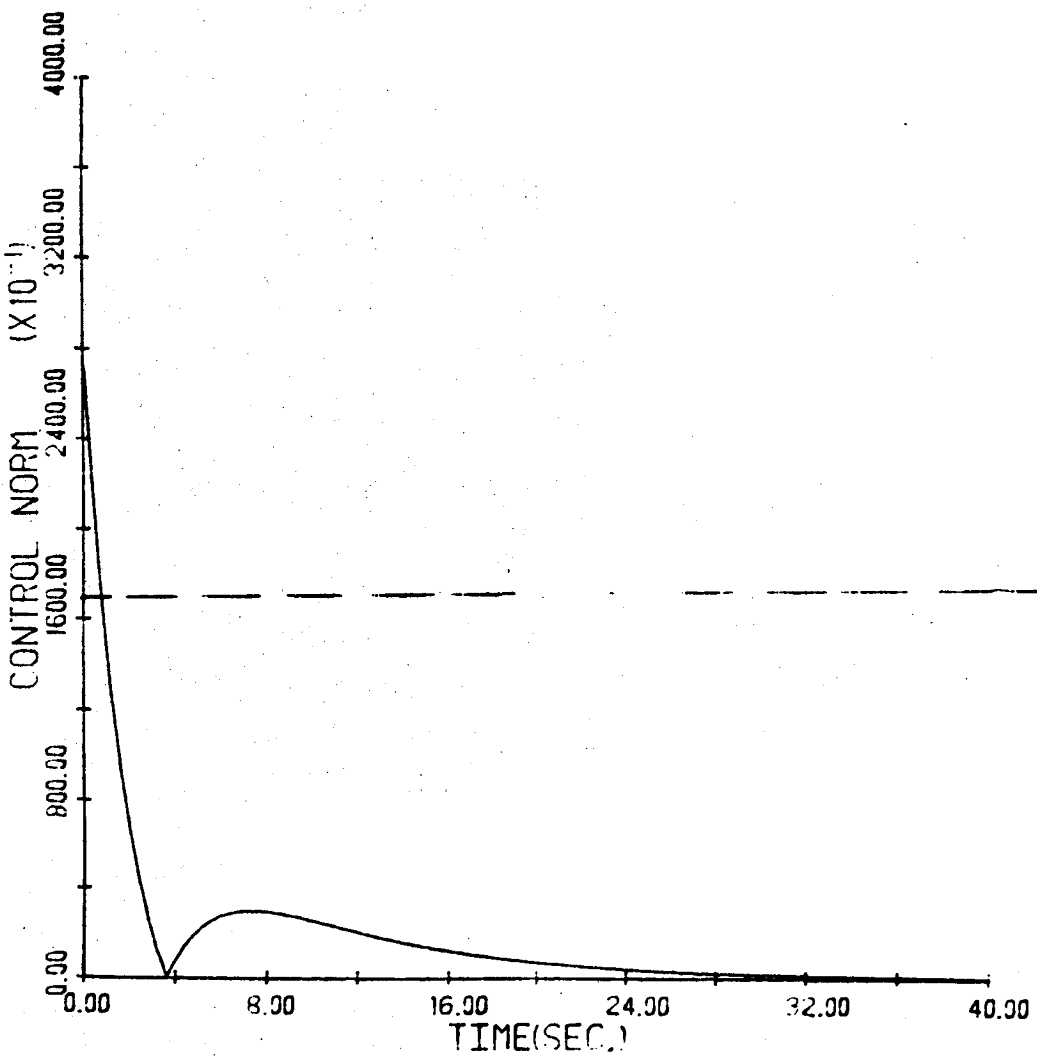


CONTROL : CASE II (a)



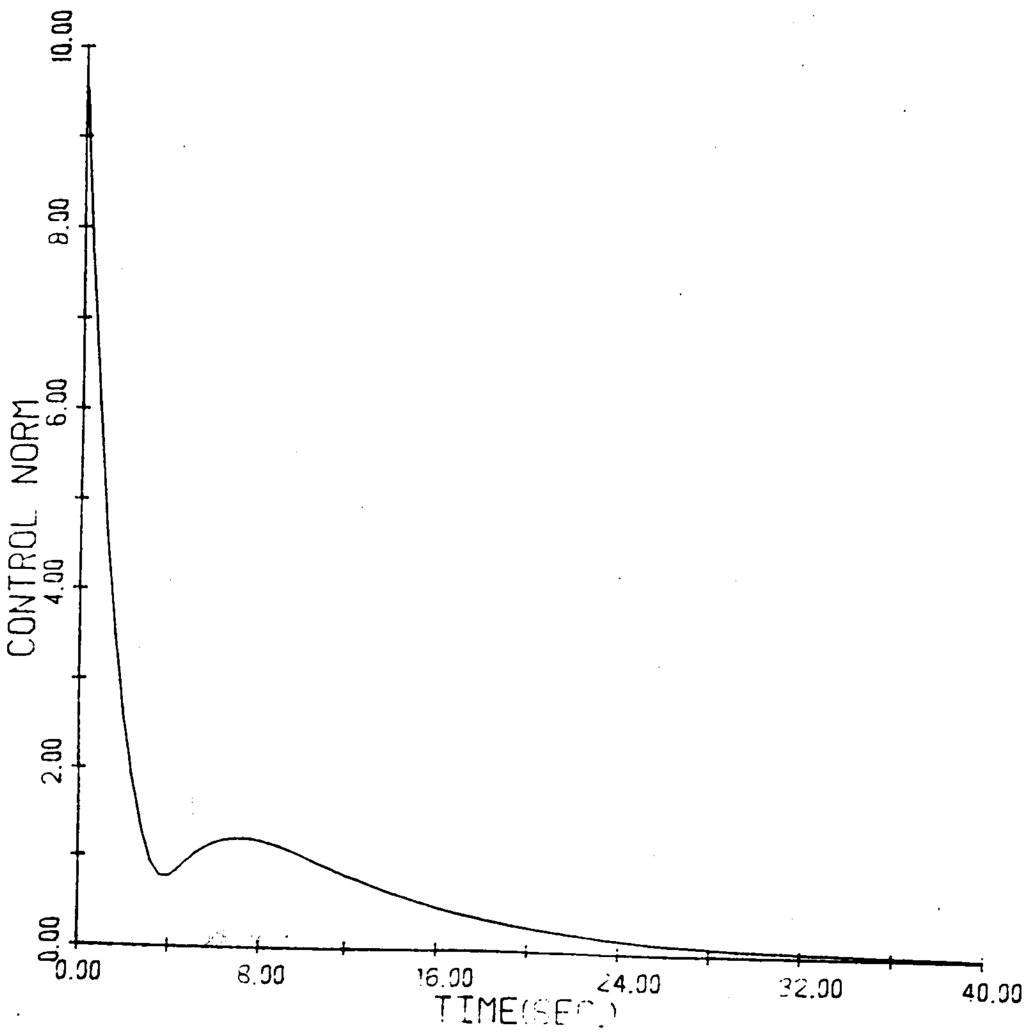
NORM OF MOMENTS AT SHUTTLE : CASE II(a)

$$= \left(T_{sx}^2 + T_{sy}^2 + T_{sz}^2\right)^{1/2}$$



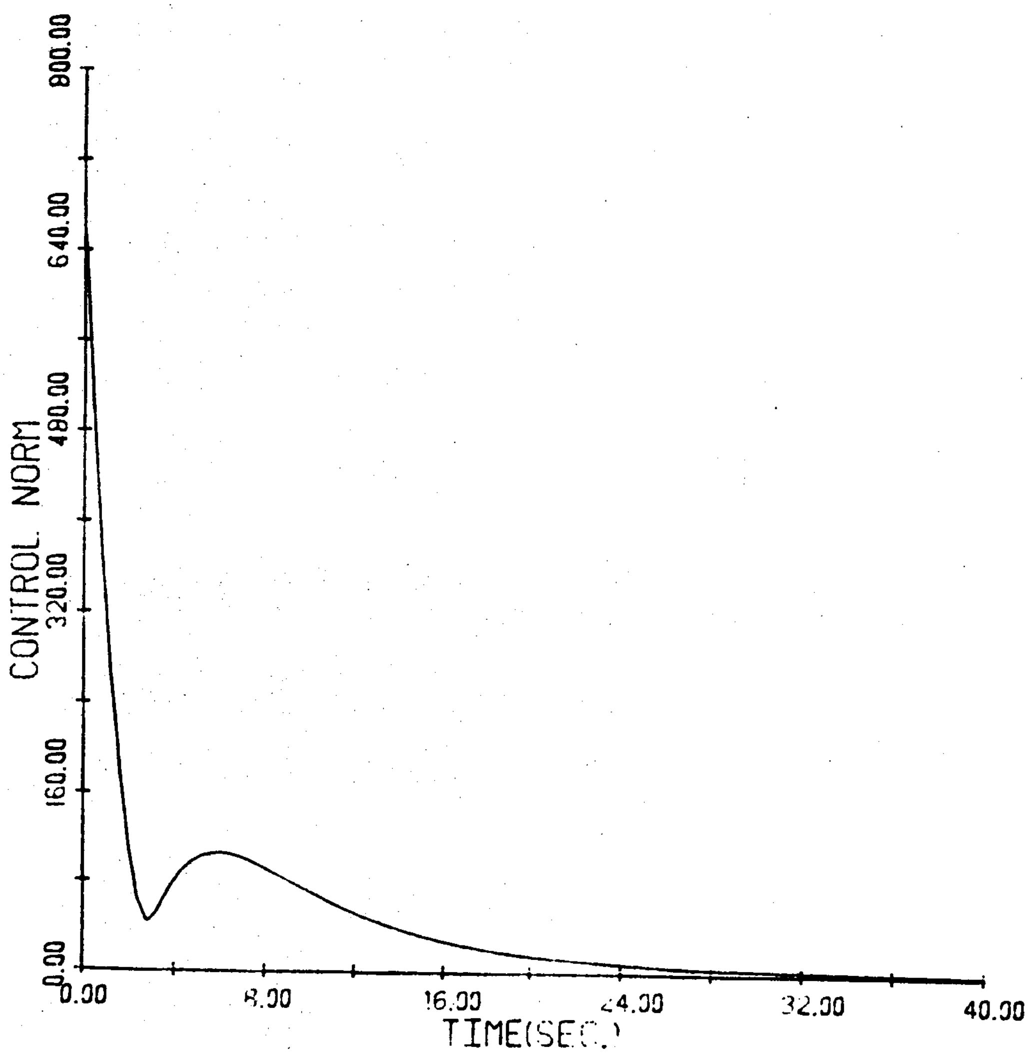
NORM of FORCES AT REFLECTOR : CASE II(a)

$$= \left(f_{rx}^2 + f_{ry}^2 \right)^{1/2}$$

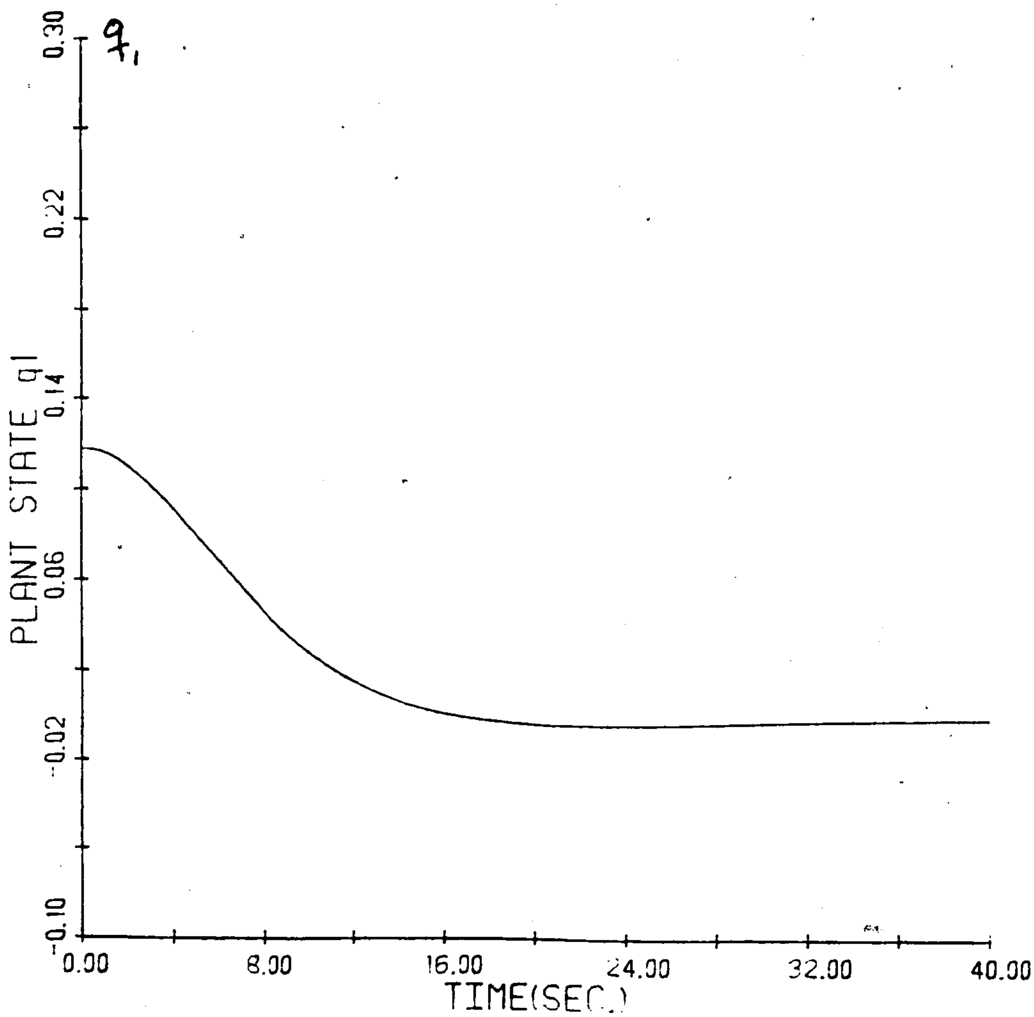


NORM of MOMENTS AT REFLECTOR : CASE II(a)

$$= \left(T_{rx}^2 + T_{ry}^2 + T_{rz}^2\right)^{1/2}$$



PLANT STATE : CASE II (a)



MODEL REFERENCE CONTROL OF DISTRIBUTED SYSTEMS

$$m(x)u_{tt}(x,t) + D_o u_t(x,t) + A_o u(x,t) = f(x,t)$$

$$v_1 = u(x,t)$$

BC

$$v_1''(0) = v_1''(L) = 0$$

$$v_2 = \frac{\partial}{\partial t} u(x,t)$$

and $v_2'''(0) = v_1'''(L) = 0$

$$\dot{\underline{v}} = A\underline{v} + B\ f(x,t)$$

$$A = \begin{bmatrix} 0 & 1 \\ \frac{-A_o}{m(x)} & \frac{-D_o}{m(x)} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m(x)} \end{bmatrix}$$

$$y = Cv$$

CONTROL PROBLEM FORMULATION

GIVEN THE DPS, IT IS DESIRED TO FIND A FINITE DIMENSIONAL CONTROLLER SO THAT THE OUTPUT $y(t)$ "FOLLOWS" A DESIRABLE OUTPUT TRAJECTORY $y_m(t)$.

$$\dot{q} = A_m q + B_m u_m$$

$$y_m = C_m q$$

SOLUTION TO DPS MRC PROBLEM

DEFINE IDEAL STATE AND CONTROL v^*, f^*

$$\frac{\partial v^*(t)}{\partial t} = A\, v^*(t) + Bf^*(t)$$

$$y^*(t) = Cv^*(t)$$

$$y^*(t) = y_m(t) = C_m q(t)$$

$$v^*(t) = S_{11}(x)\ q(t) + S_{12}(x)\ u_m(t)$$

$$f^*(t) = S_{21}\ q(t) + S_{22}\ u_m(t)$$

$$y^*(t) = C\ S_{11}\ q + C\ S_{12} u_m = y_m = C_m\ q$$

$$S_{11}(x)\ A_m = A\ S_{11}(x) + B\ S_{21}$$

$$S_{11} B_m = A\ S_{12}(x) + B\ S_{22}$$

$$C\ S_{11} = C_m$$

$$C\ S_{12} = 0$$

$$\dot{e} = \dot{v}^* - \dot{v} =$$

$$Av^* + Bf^* - Av - Bf =$$

$$Ae + B\,(f^* - f)$$

THIS EQUATION SUGGESTS THAT THE ACTUAL MODEL FOLLOWING CONTROL (f) BE DEFINED AS:

$$f = f^* + G(y_m - y)$$

$$= f^* + G(C\,q_m - Cv)$$

$$= f^* + G\,C(v^* - v)$$

$$= f^* + G\,C\,e$$

SUBSTITUTION OF (3.14) INTO (3.13) GIVES:

$$\dot{e} = (A - B\,G\,C)e$$

FOR ILLUSTRATIVE PURPOSES WE WILL CONTROL

$$\omega_4 = \dot{v}'(t,L) = y_1$$

$$\omega_1 = \dot{v}'(t,0) = y_2$$

AND

$$y_3 = v(t,s_o) + \alpha\dot{v}(t,s_o)$$

WHERE

$$0 < s_o < L$$

THUS

$$C = \begin{bmatrix} 0 & \partial/\partial s[\delta(s-L)] \\ 0 & \frac{\partial}{\partial s}\,\delta(s) \\ \delta(s-s_o) & \alpha\delta(s-s_o) \end{bmatrix}$$

REFERENCE MODEL

$$\dot{q} = a_m q + b_m u_m$$

$$y_{m1} = c_1 q$$

$$y_{m2} = c_2 q$$

$$y_{m3} = c_3 q$$

UNDER THE ASSUMPTION THAT $u_m = 0$,

$$v_1^* = S_{11}{}^1(s)\ q(t)$$

$$v_2^* = S_{11}{}^2(s)\ q(t)$$

$$f_1^* = S_{21}{}^1\ q$$

$$f_2^* = S_{21}{}^2\ q$$

$$f_3^* = S_{21}{}^3\ q$$

ILLUSTRATIVE APPLICATION TO SCOLE

ROLL BEAM BENDING EQUATION

ASSUMPTIONS: PROOF MASSES AND DAMPING NEGLIGIBLE, REFLECTION MASS NEGLIGIBLE

$$PA\,\ddot{v}(t,s) + EI\,v''''(t,s) = f_1\ \delta(s-1) + f_2\ \delta'(s) + f_3\ \delta'(s-L)$$

$$f_i = F_y$$

$$f_2 = M_i$$

$$f_3 \quad M_y$$

CASE 1: (ignore shuttle mass)

$$v''(t,0) = v''(t,L) = 0$$

$$v'''(t,0) = v'''(t,L) = 0$$

CASE 2: Simple-free (shuttle mass $\longrightarrow$ infinity)

$$v(t,0) = 0 \qquad v'(0,t) = 0$$

$$v''(t,L) = 0 \qquad v'''(t,L) = 0$$

CASE 1 Free-Free

$$S_{11}^{\;1}(s) =$$

$$+ S_{21}^{\;1} \sum_K \frac{X_K(L)}{\frac{L}{4}\left(PA\,a_m^{\;2} + EIK^4\right) X_K^2(L)} X_K'(s)$$

$$+ S_{21}^{\;2} \sum_K \frac{-X_K'(0)}{\frac{L}{4}\left(PA\,a_m^{\;2} + EIK^4\right) X_K^2(L)} X_K'(s)$$

$$+ S_{21}^{\;3} \sum_K \frac{-X_K'(L)}{\frac{L}{4}\left(PA\,a_m^{\;2} + EIK^4\right) X_K^2(L)} X_K'(s)$$

WHERE

$$X_K(s) = \left[\left(\frac{\mathrm{Sinh}(KL) - \mathrm{Sin}(KL)}{\mathrm{Cos}(KL) - \mathrm{Cosh}(KL)} (\mathrm{Cosh}\,Ks + \mathrm{Cos}\,Ks) \right) + \mathrm{Sinh}(Ks) + \mathrm{Sin}(Ks) \right]$$

SOLVING THE EQUATIONS OF MOTION

$$v(t,s) = \sum_{n-1}^{\infty} Q_n(s) y_n(t)$$

$$EI \frac{d^4 Q_n(s)}{ds^4} - W_n^2 \, PA \, Q_n(s) = \emptyset \qquad n = 1,2,\ldots\infty$$

$$Q_n(s) = A \sin k_n s + B \cos K_n s + C \sinh k_n s$$
$$+ D \cosh K_n s$$

CASE 1

$$Q_n(s) = \left[\frac{\sinh k_nL - \sin k_nL}{\cos k_nL - \cosh k_nL}\right](\cosh k_ns + \cos k_ns)$$

$$+ \sin k_ns + \sinh k_ns$$

CASE

$$Q_n(s) = \frac{1}{N_n}\left[\frac{\sin k_nL + \sinh k_nL}{\cos k_nL + \cosh k_nL}\right](\cos k_ns - \cosh k_ns)$$

$$+ \sin k_ns \quad \sinh k_ns$$

CGT GAIN SOLUTION

Output matrix values

Note: $S_{11}(0) = \frac{C_1}{A_m}$

$$S_{11}(L) = \frac{C_2}{A_m}$$

$$S_{11}(S\ ) = \frac{C_3}{1 + \alpha A_m}$$

Want $S_{11}(s)$ to be ideal initial beam shape

Case 1	Case 2
C_1 = 0.00251	C_1 = 0.002
C_2 = - 0.00251	C_2 = - 0.001456
C_3 = 0.09596	C_3 = 0.092

REFERENCE MODEL SELECTION

PURPOSE: TO DAMP OUT THE STRUCTURAL VIBRATIONS WITHIN TEN SECONDS WITHOUT VIOLATING THE CONTROL MAGNITUDE CONSTRAINTS.

$$\dot{q}(t) = 0.4q(t)$$

$$y_{m1}(t) = C_{m1}q(t)$$

$$y_{m2}(t) = C_{m2}q(t)$$

$$y_{m3}(t) = C_{m3}q(t)$$

$$q(0) = 1$$

RESULTS

- FEEDBACK GAINS
- FEEDFORWARD GAINS
- SIMULATIONS

PARAMETER	VALUE
L = Beam Length	130.0 ft
α = weighting factor	0.25
s = additional beam sensor location	65.0 ft
EI	4.0×10^7 lb-ft^2
PA	0.09556 slugs/ft

TABLE 5.1 : SCOLE BEAM PARAMETERS

MODE # n	k_n	W_n (rad/sec)	f_n (HZ)
1	4.73	27.085	4.311
2	7.853	74.658	11.882
3	10.996	146.378	23.297
4	14.173	243.181	38.703
5	17.274	361.236	57.492

TABLE 5.2 : NATURAL FREQUENCIES FOR CASE I

MODE # n	k_n	W_n (rad/sec)	f_n (Hz)
1	3.927	18.669	2.971
2	7.069	60.495	9.628
3	10.210	126.199	20.085
4	13.352	215.823	34.349
5	16.493	329.310	52.411

TABLE 5.3 : NATURAL FREQUENCIES FOR CASE II

RECALL THAT FOR TRUE STABILITY WE NEED

$$\underline{f} = f^* + G(y_m - y) = f^* + GC(v^* - v)$$

THIS SYSTEM WILL BE STABLE FOR

$$G = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{bmatrix}$$

$$G_{ii} > 0$$

FEEDBACK GAINS (NO SENSOR AT S_0)

CASE I

SET A : $G_{11} = 1210.0$ $G_{22} = 1730.0$ $G_{33} = 0.0$

SET B : $G_{11} = 600.0$ $G_{22} = 850.0$ $G_{33} = 0.0$

SET C : $G_{11} = 60.0$ $G_{22} = 85.0$ $G_{33} = 0.0$

CASE II

SET A : $G_{11} = 950.0$ $G_{22} = 300.0$ $G_{33} = 0.0$

SET B : $G_{11} = 475.0$ $G_{22} = 150.0$ $G_{33} = 0.0$

SET C : $G_{11} = 95.0$ $G_{22} = 30.0$ $G_{33} = 0.0$

# modes in series	S_{21}^{1}	S_{21}^{2}	S_{21}^{3}
3	-690.946	-1641.531	-9785.292
4	-690.954	-1044.241	-9188.201
5	-669.460	-1236.139	-8678.003
6	-669.401	-949.532	-8391.477
7	-641.403	-1145.781	-7803.666
8	-641.402	-982.676	-7640.603
9	-631.98	-1066.760	-7320.688
10	-631.916	-959.505	-8279.688
AVERAGE	-658.000	-1127.700	-8279.000

CGT Gains for Case I

# modes in series	S_{21}^{1}	S_{21}^{2}	S_{21}^{3}
3	-355.878	-1141.894	-6309.016
4	-332.129	-805.507	-5354.048
5	-318.384	-1007.015	-4986.424
6	-316.305	-791.389	-4776.447
7	-303.601	-953.259	-4463.880
8	-300.262	-856.221	-4300.704
9	-295.713	-924.748	-4184.084
10	-294.991	-937.512	-4166.822
AVERAGE	-314.235	-926.750	-4817.250

[redacted]

Simulations

Case I : Free-Free

$G_{11} = 1210$ $G_{22} = 1731$ $G_{33} = 0$

$S_{21} = -669.0$ $S_{22} = -949.5$ $S_{23} = -8391.5$

Case II : Simple-Free

$G_{11} = 950$ $G_{22} = 300$ $G_{33} = 0$

$S_{21} = -316.3$ $S_{22} = -791.4$ $S_{23} = -4776.4$

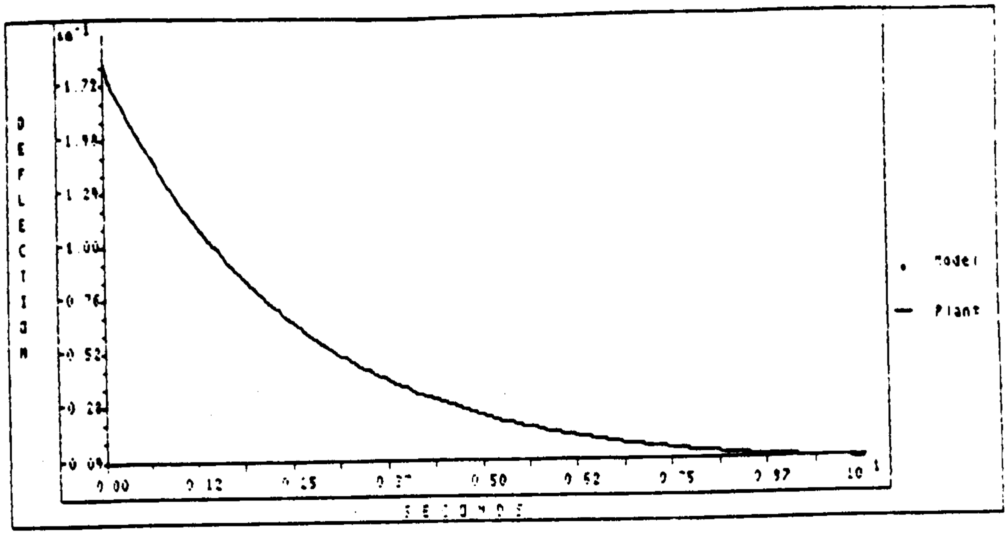


Figure 5.11 Case I : Perfect Model Following-
CGT Gains for 6 Modes

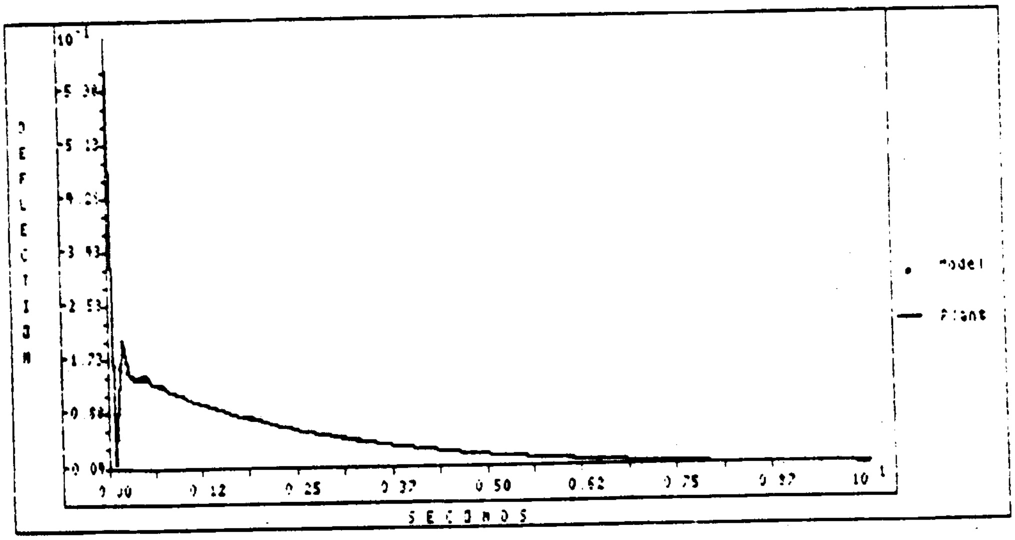


Figure 5.12 Case I : Tracking -
CGT Gains for 4 Modes

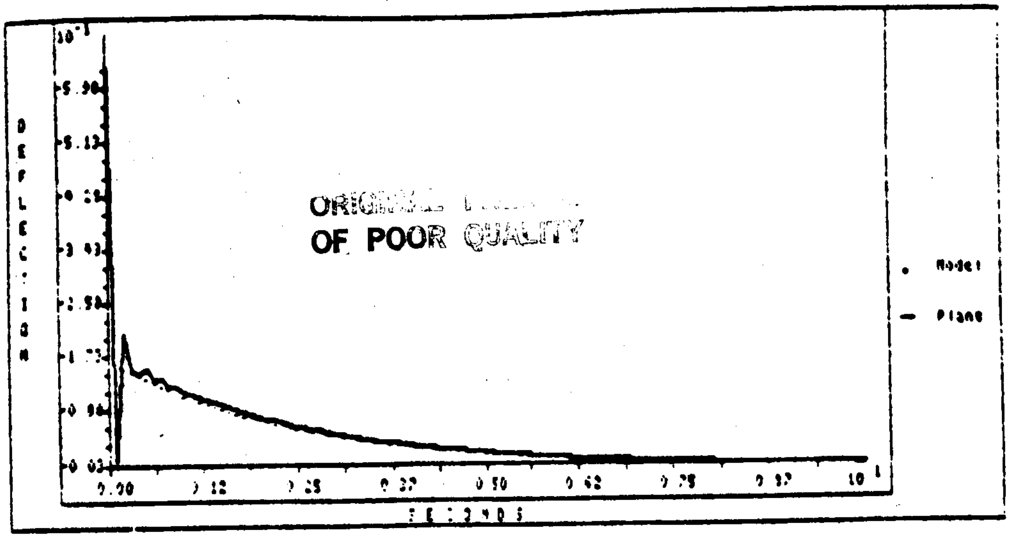


Figure 5.13 Case I : Tracking -
CGT Gains for 6 Modes

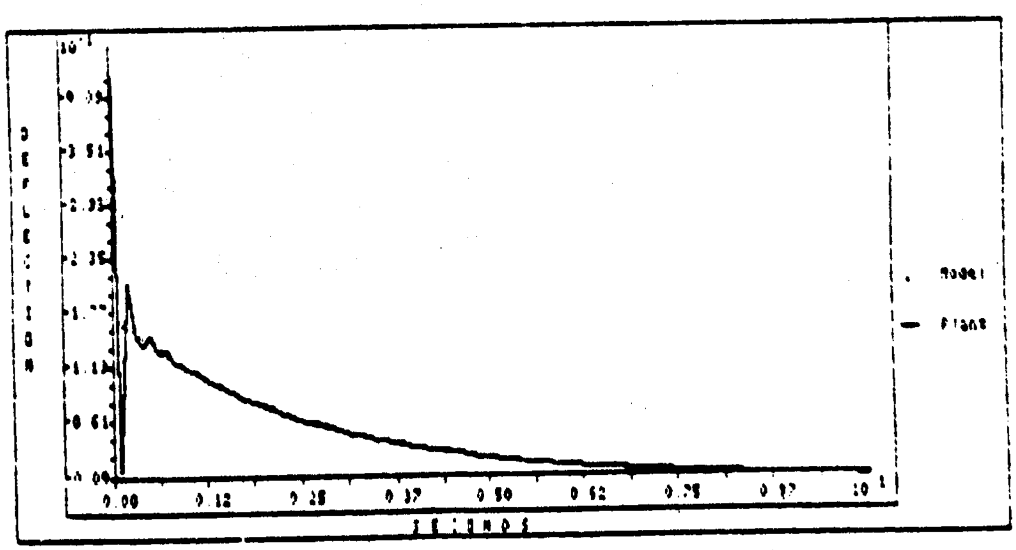


Figure 5.14 Case I : Tracking -
CGT Gains for Ave Modes

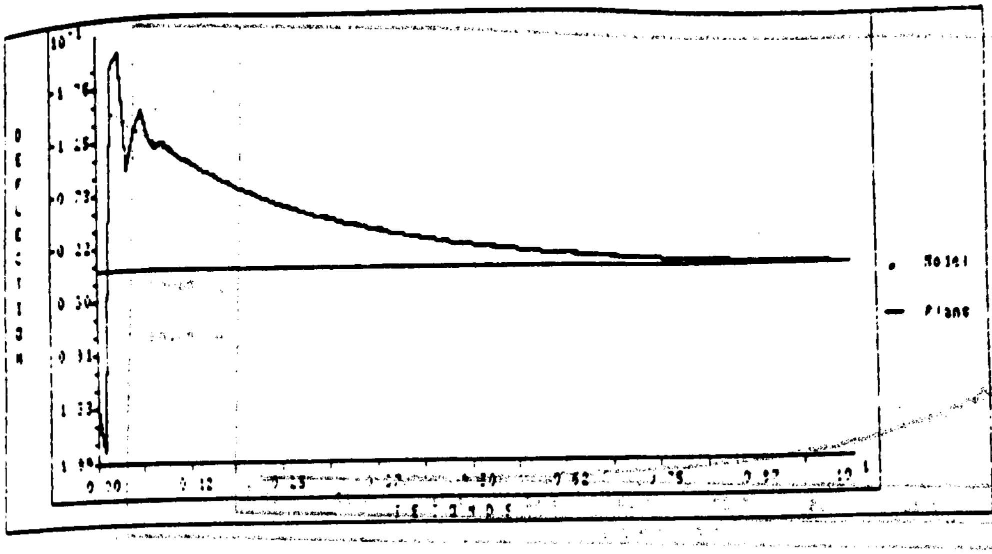


Figure 5.15 Case II : Tracking -
CGT Gains for 4 Modes

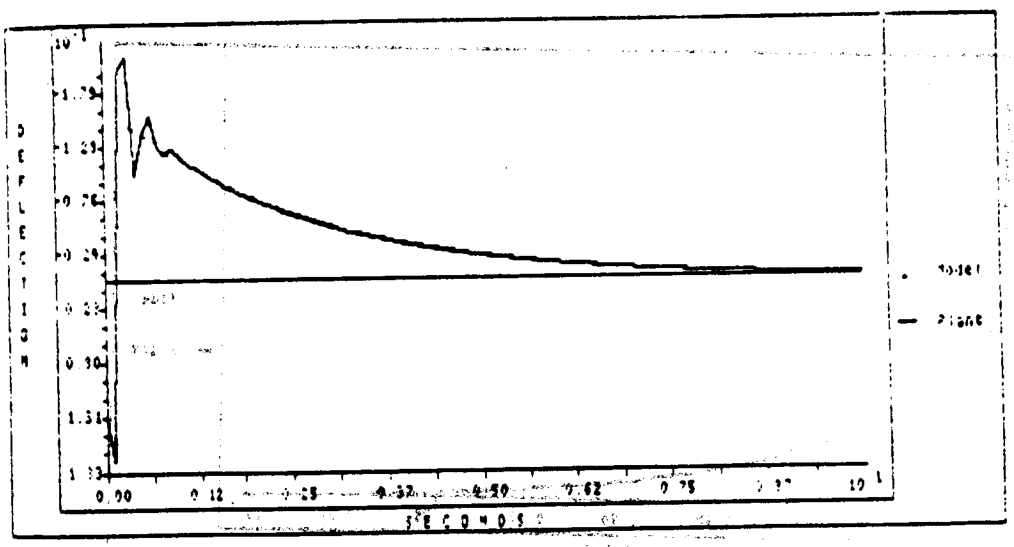


Figure 5.16 Case II : Tracking -
CGT Gains for 6 Modes

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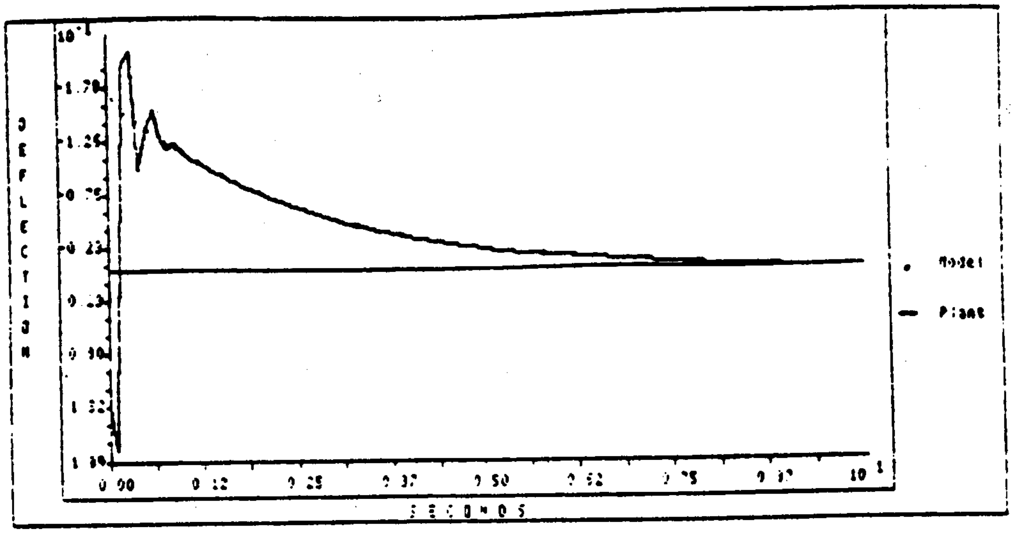

Figure 5.17 Case II : Tracking -
CGT Gains for Ave Modes

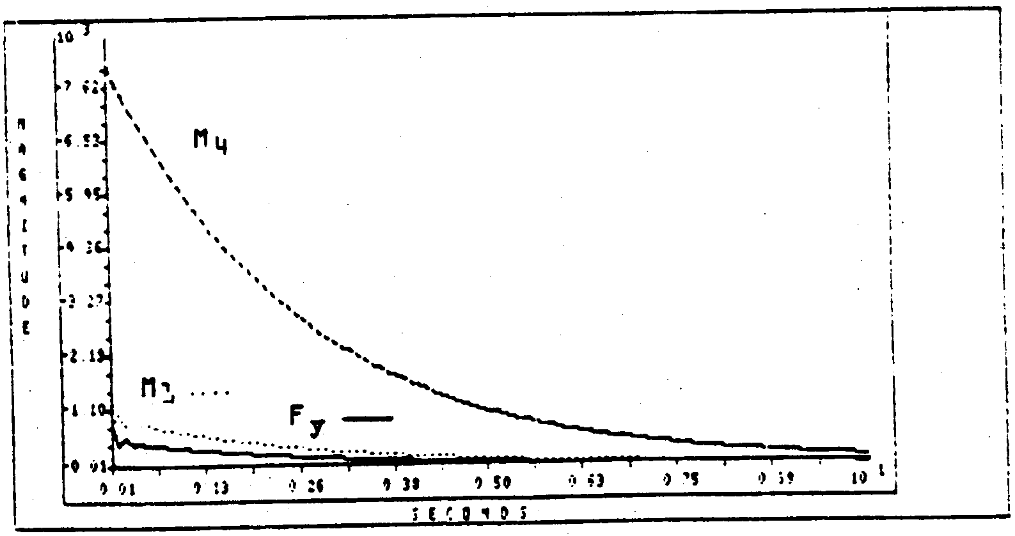

Figure 5.18 Case I : Controls -
CGT Gains for 6 Modes

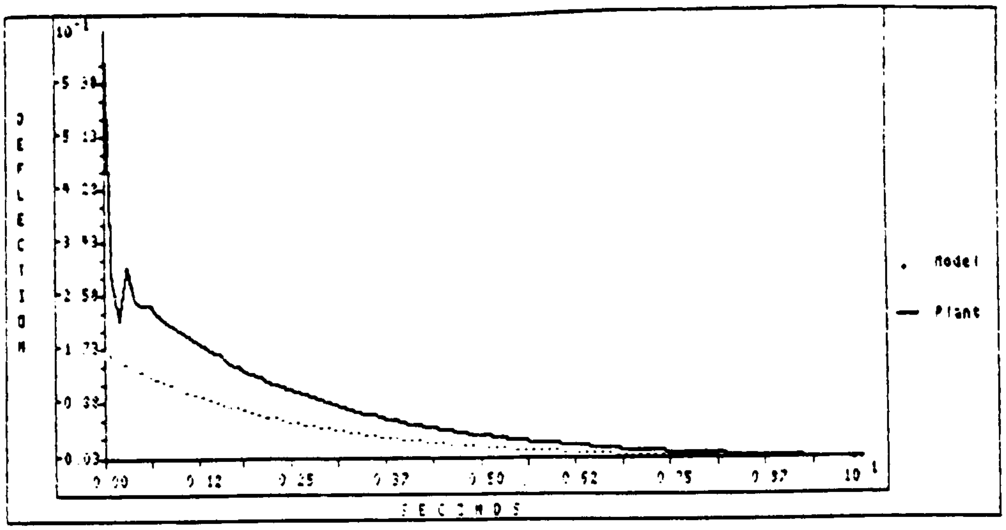

Figure 5.19 Case I : Parameter Variation - CGT Gains for 6 Modes

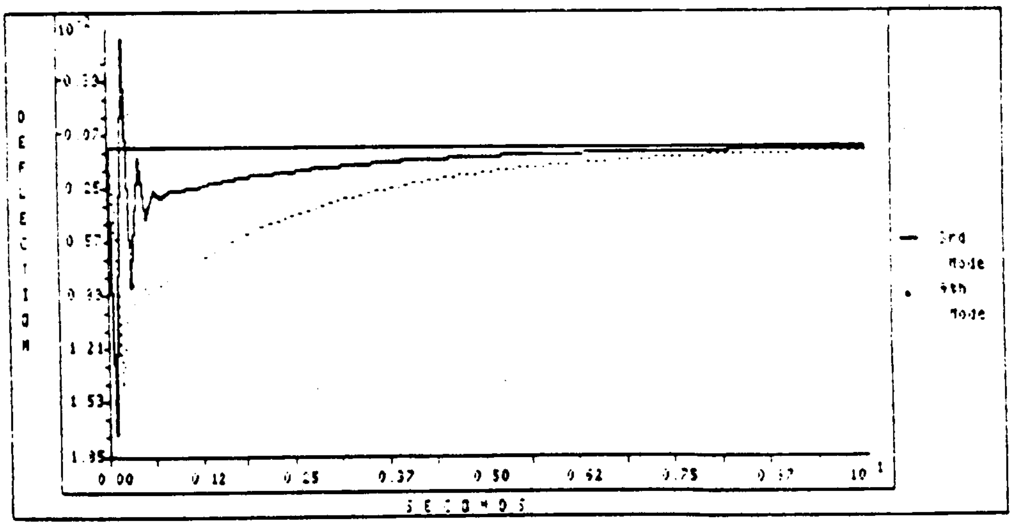

Figure 5.20 Case I : Control Spillover - Beam Deflection for 3rd and 4th Modes, CGT Gains for 6 Modes

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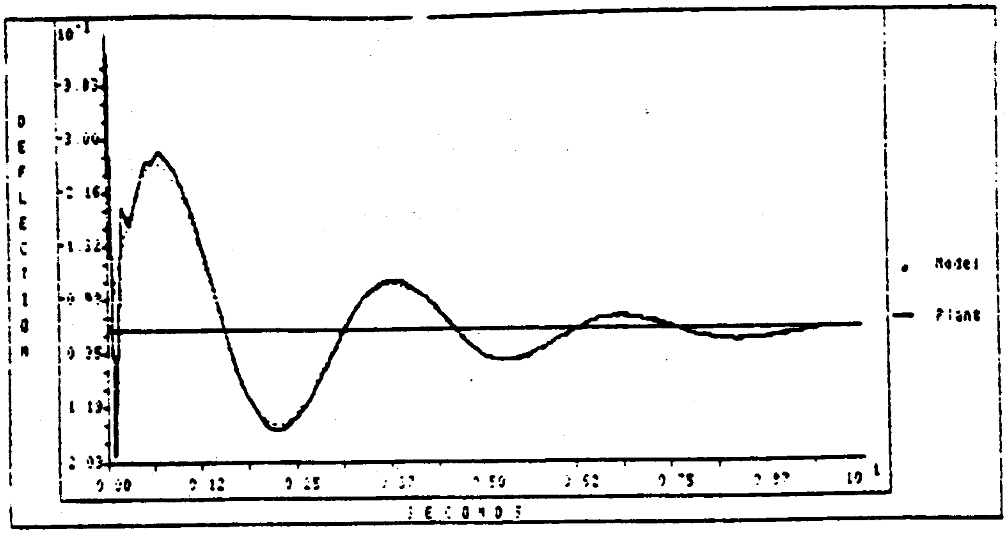

Figure 5.21 Case I : Tracking -
Reference Model 2.04 $e^{-.4t}$ sin (2.04t),
CGT Gains for 6 Modes

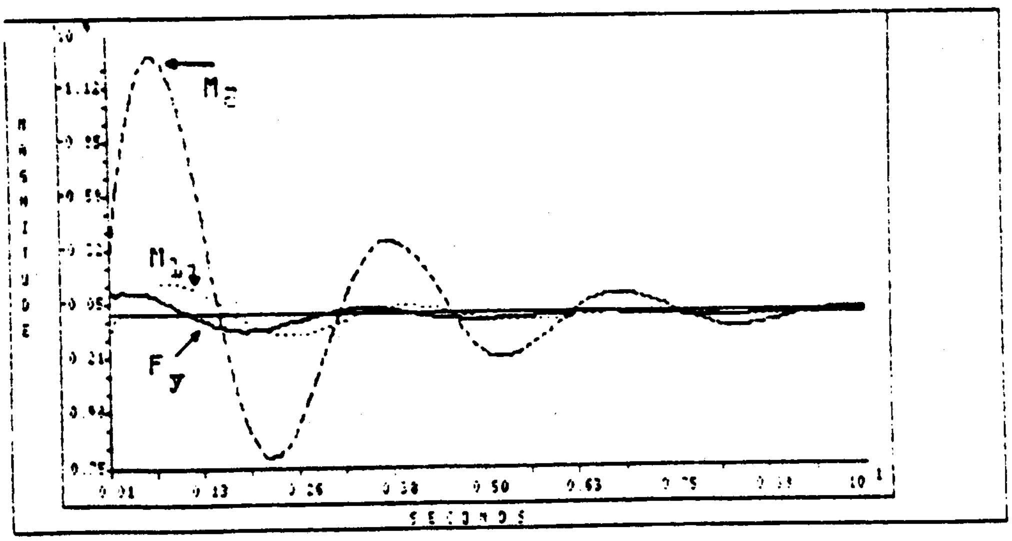

Figure 5.22 Case I : Controls -
Reference Model 2.04 $e^{-.4t}$ sin (2.04t),
CGT Gains for 6 Modes

CONCLUSIONS

1. A MODEL FOLLOWING PROCEDURE USING CGT THEORY WAS DEVELOPED FOR APPLICATION TO DPS SYSTEMS.

2. THE DESIGN PROCEDURE RESULTS IN A FINITE-DIMENSIONAL CONTROLLER THAT GIVES OUTPUT FOLLOWING AND FULL STATE STABILITY.

3. THE MODEL FOLLOWING CONTROLLER'S APPLICATION TO A MODELS OF THE SCOLE WAS SHOWN.

4. THROUGH SIMULATIONS, IT WAS DEMONSTRATED THAT SATISFACTORY TRACKING OF A DESIRED TRAJECTORY CAN BE ACHIEVED.

5. EXCITATION OF HIGHER ORDER MODES BY THE CONTROLLER AND PARAMETER VARIATIONS DO NOT ADVERSELY AFFECT THE SYSTEM PERFORMANCE.

PLANNED ACTIVITIES

- o DEVELOPMENT OF OUTPUT FEEDBACK FOR LUMPED MODEL CONTROLLER
- o DEVELOPMENT OF CONTROLS FOR THE PITCH AND YAW TORSION EQUATION
- o TESTING OF THE CONTROLLERS, IN THE PRESENCE OF NOISY SENSORS.
- o ADAPTIVE CONTROL